見微知著: 數學居然算的出大腦

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Brain and Learning Lab.

數學能力的神經機制

- 大腦如何進行數學計算?
- 大腦如何發展出數學能力？
- 非典型發展兒童的腦如何進行計算?
- 大腦的計算能力是否得以治療或是復健?

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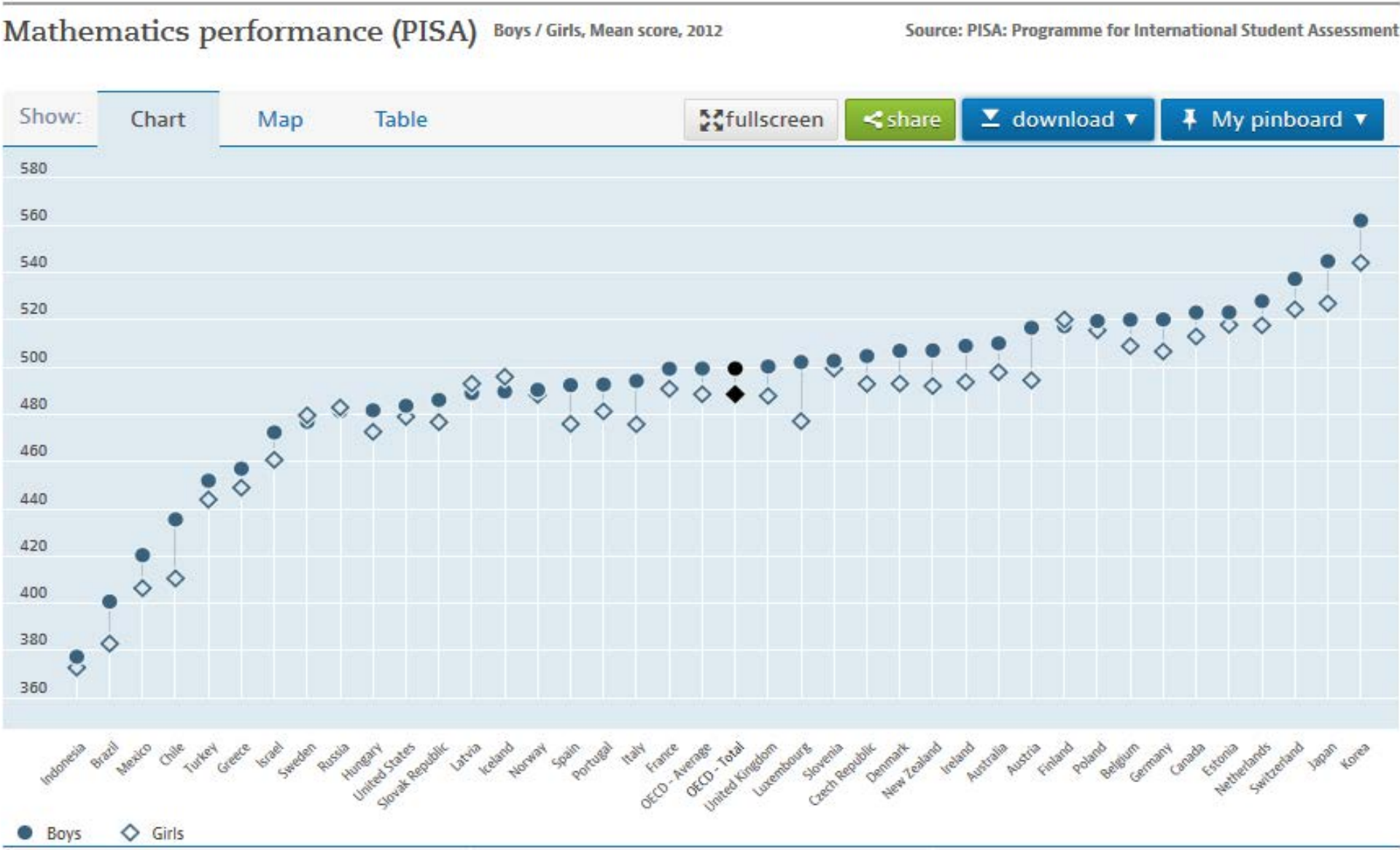
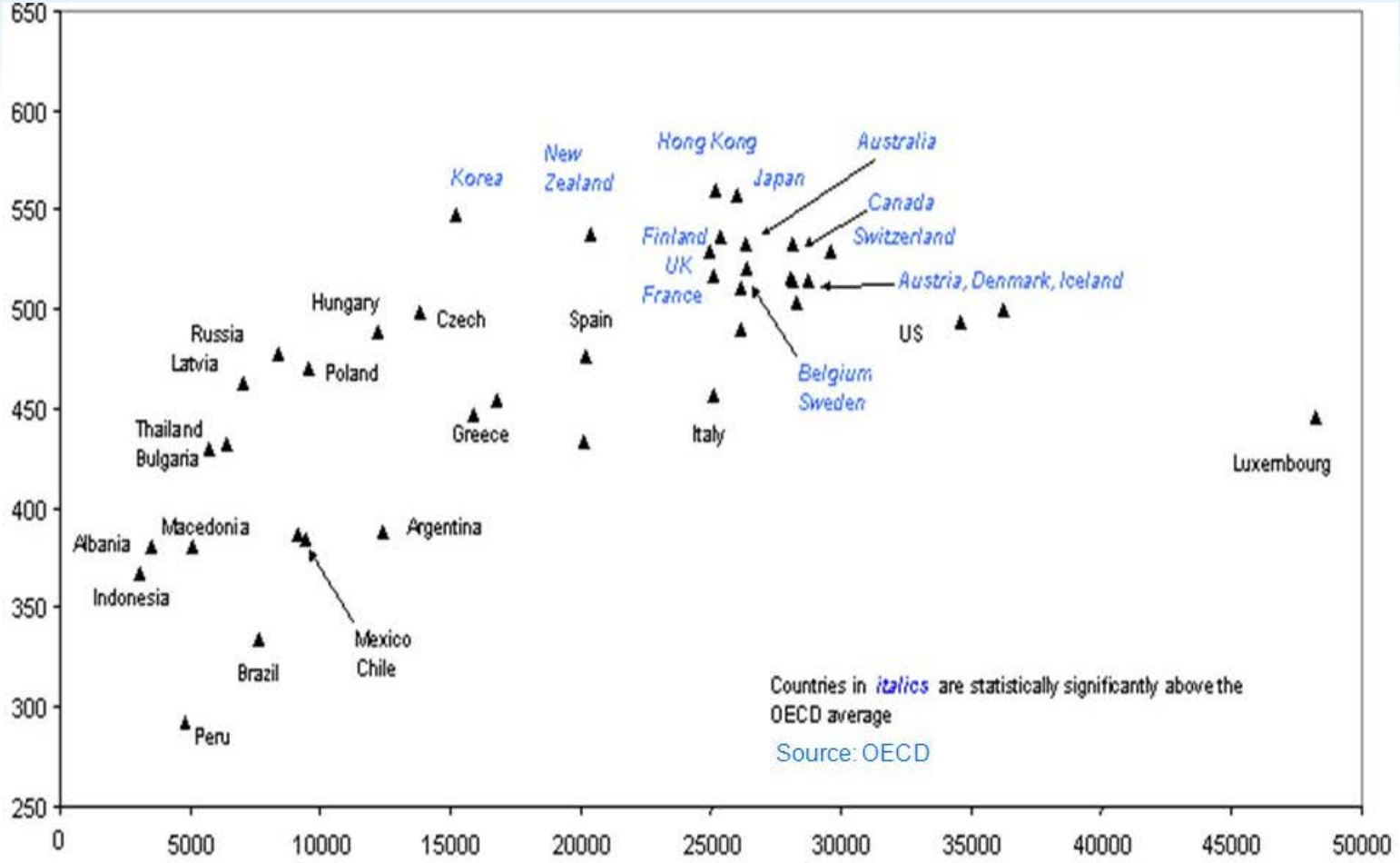
數學能力之國際評比



(source: from OECD)

Students math performance and GDP per capita



"0.5 SD in mathematics performance at the individual level implies an increase in annual growth rates of GDP per capita of 0.87%"

~OECD (2010)

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台北市某高中課表

節	時間	星期一	星期二	星期三	星期四	星期五
1	0810-0900	班週會	歷史	國文	國文	英文
2	0910-1000	英文	基礎地科上	體育	數學	體育
3	1010-1100	家政	特色課程	國防通識	英文	基礎化學
4	1110-1200	家政	特色課程	數學	地理	基礎化學
5	1310-1400	國文	公民與社會	美術	資訊概論	數學
6	1410-1500	地理	公民與社會	英文	資訊 / 生涯	數學
7	1520-1610	音樂	數學.輔	歷史	基礎地科上	國文

各科節數對照：

科目	國文	數學	英文	歷史	地理	化學	地科	公民	資/生	國防	美/音	家政	體育
節數	4	5	4	2	2	2	2	2	2	1	2	2	2

What is mathematical skill?

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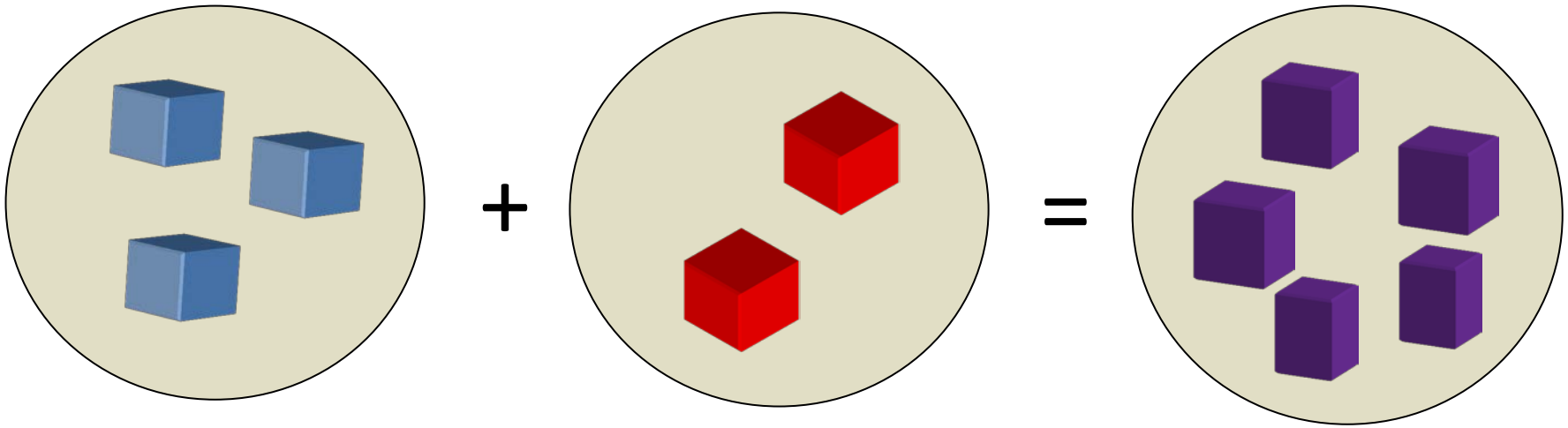
Arithmetic skill



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How does human brain process arithmetic?

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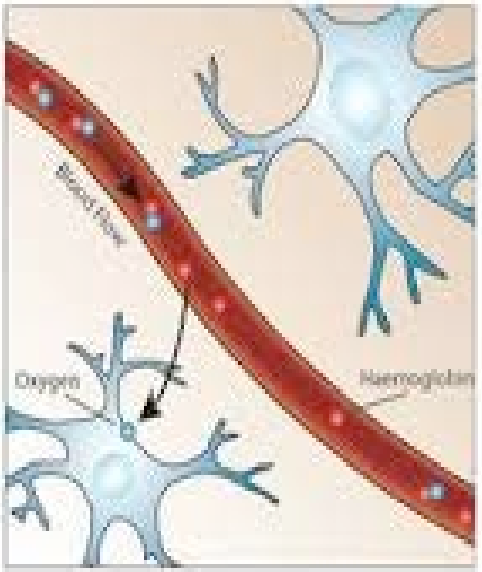
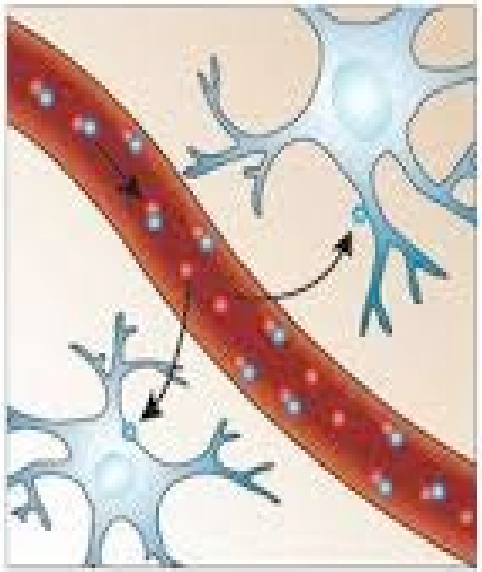
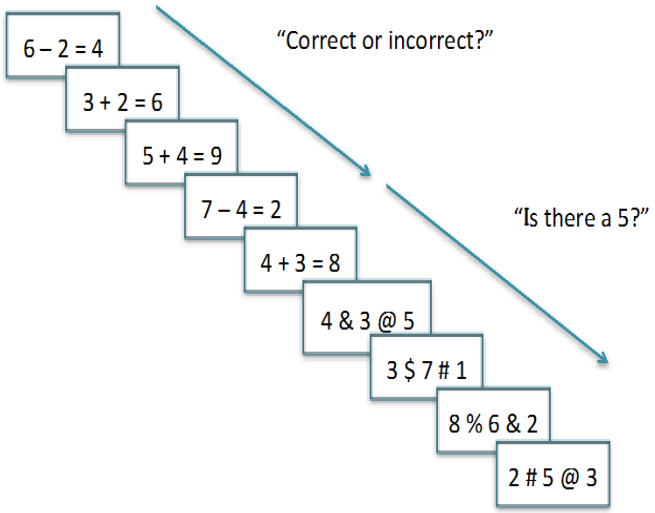
functional Magnetic Resonance Imaging



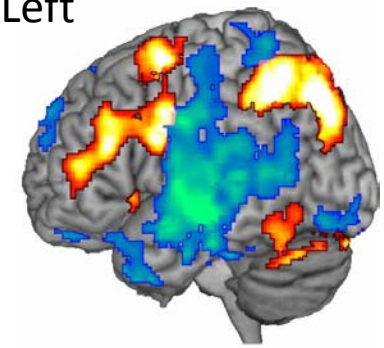
Resting



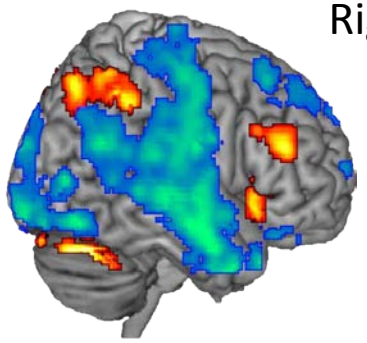
Activated



Left



Right



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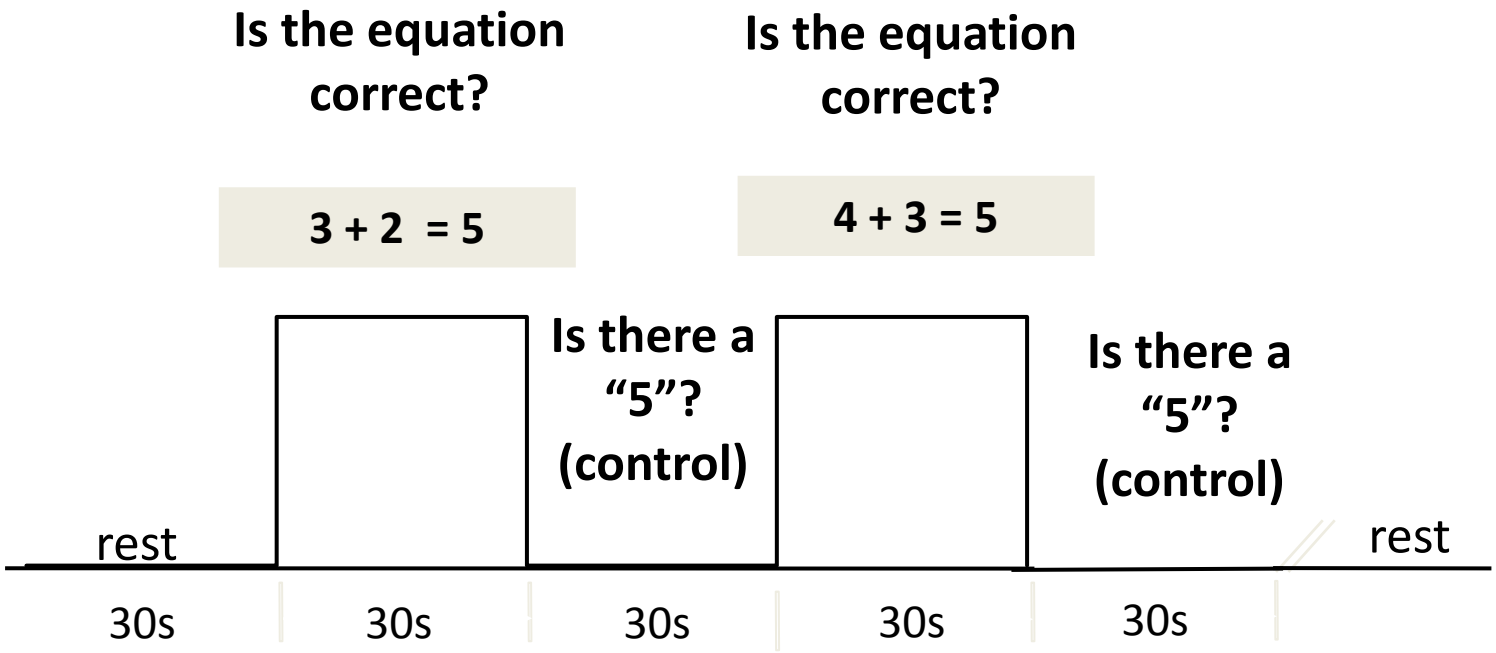
Equation verification task



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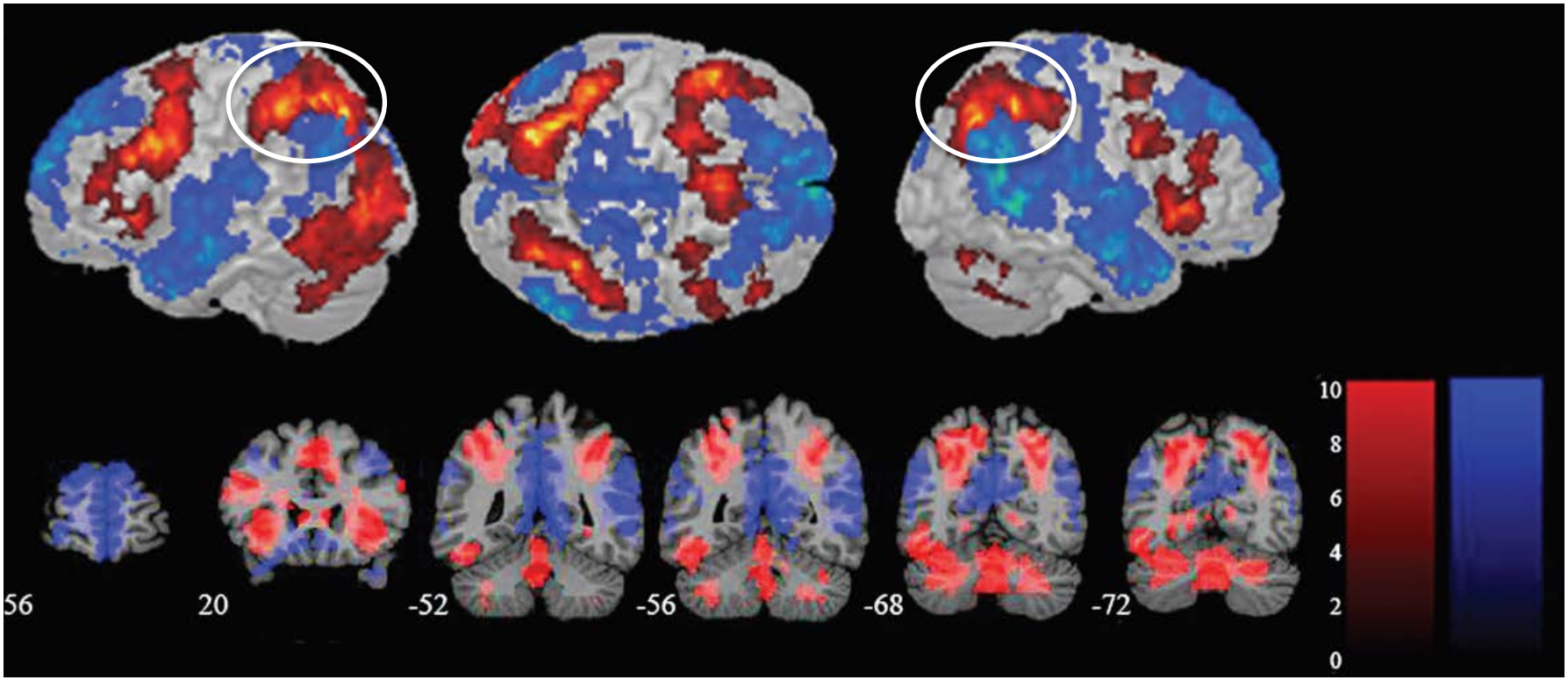
Brain regions associated with calculation



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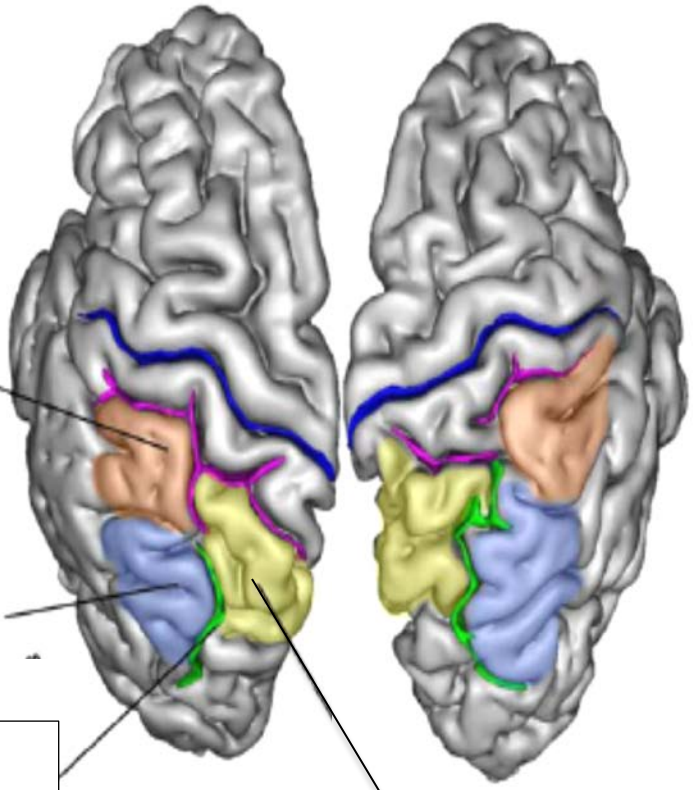
Wu, Chang et al. (2009) *Cereb Cx*

Posterior Parietal Cortex (PPC)



Supramarginal Gyrus (SMG):
Visual and Phonological Working Memory

Angular Gyrus (AG):
Verbal Fact Retrieval

Intraparietal Sulcus (IPS):
Domain Specific Abstract Quantity Representation

Superior Parietal Lobule (SPL):
Attentional Orientation

Menon (2014), Dehaene et al., (2003)

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Are the neural correlates of mental arithmetic modulated by mathematical competence?

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- Screened a large sample of adults (138)

- Selected *individuals who did not differ in IQ but varied in their mathematical competence*

- fMRI study

  - Multiplication verification **4 x 6 = 24**

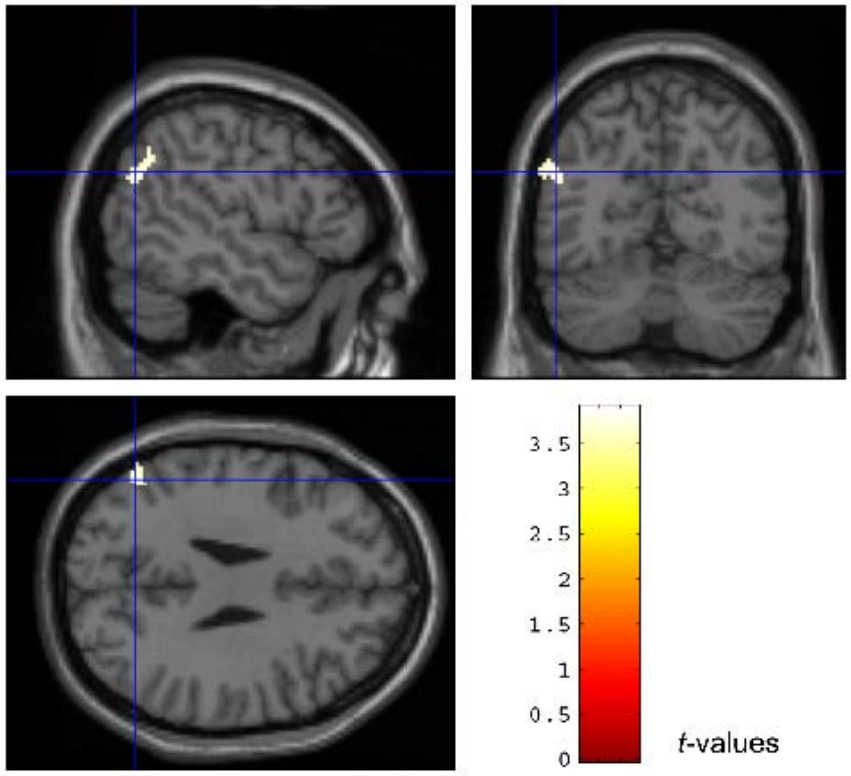
  - Control Task **3 = 3 = 3**

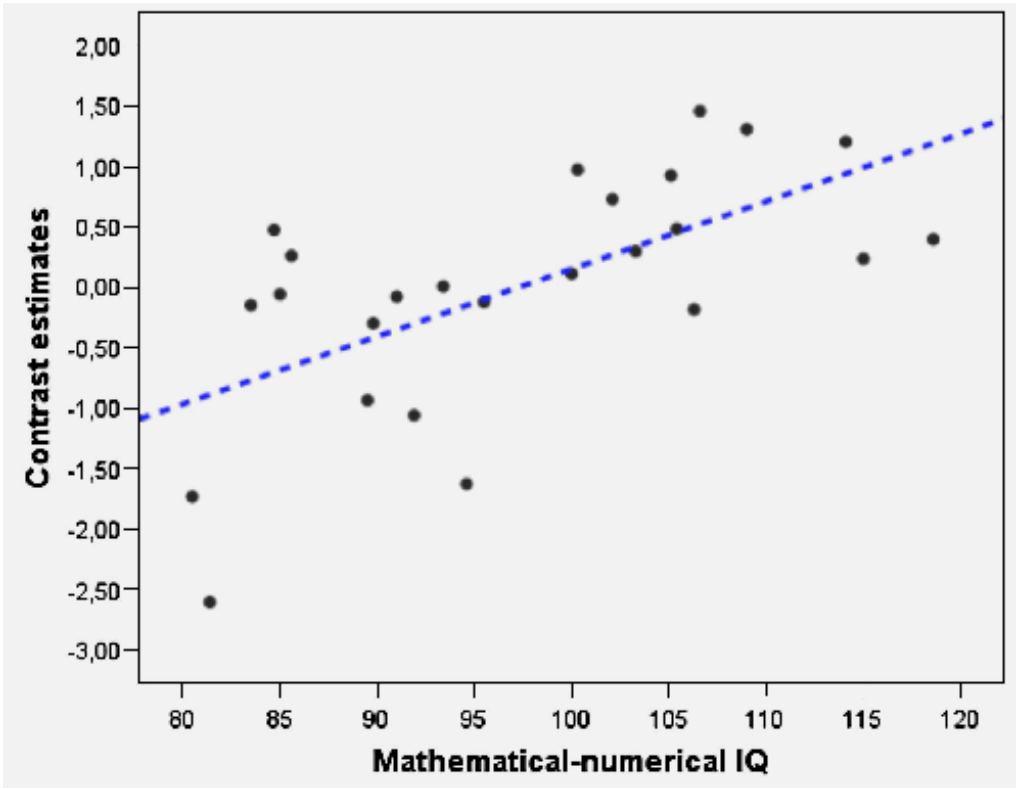
Which brain regions activated during multiplication correlated with mathematical competence?

Brain and Learning Lab.

Grabner, Ansari et al. (2007)

Relationship between AG activation and individual difference in math skill



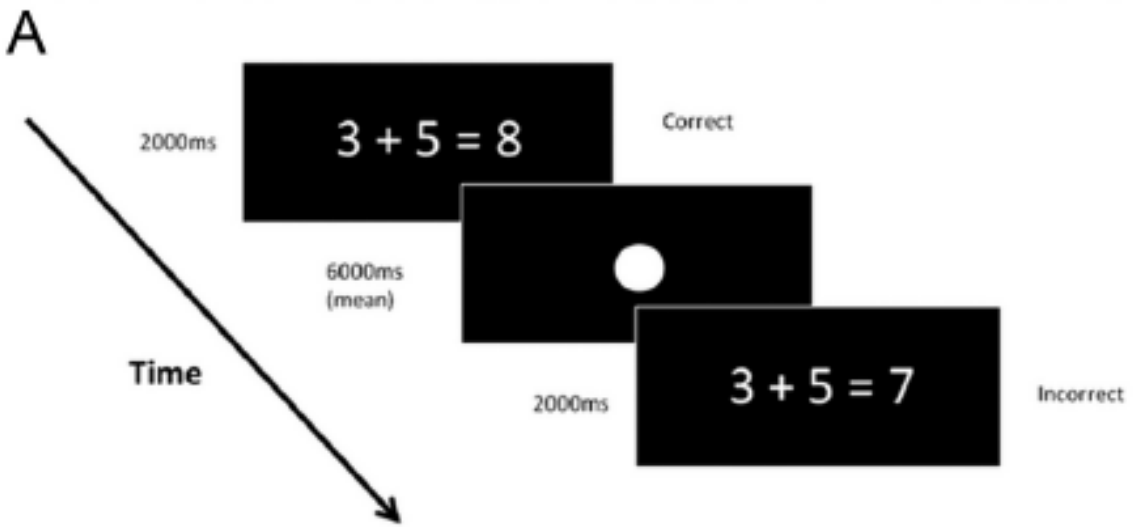


Brain and Learning Lab.

Grabner, Ansari et al. (2007)

Brain activation predicts high school math

- Participants
  - 33 high school students (mean age :17 yrs)
- Math skill assessment
  - PSAT



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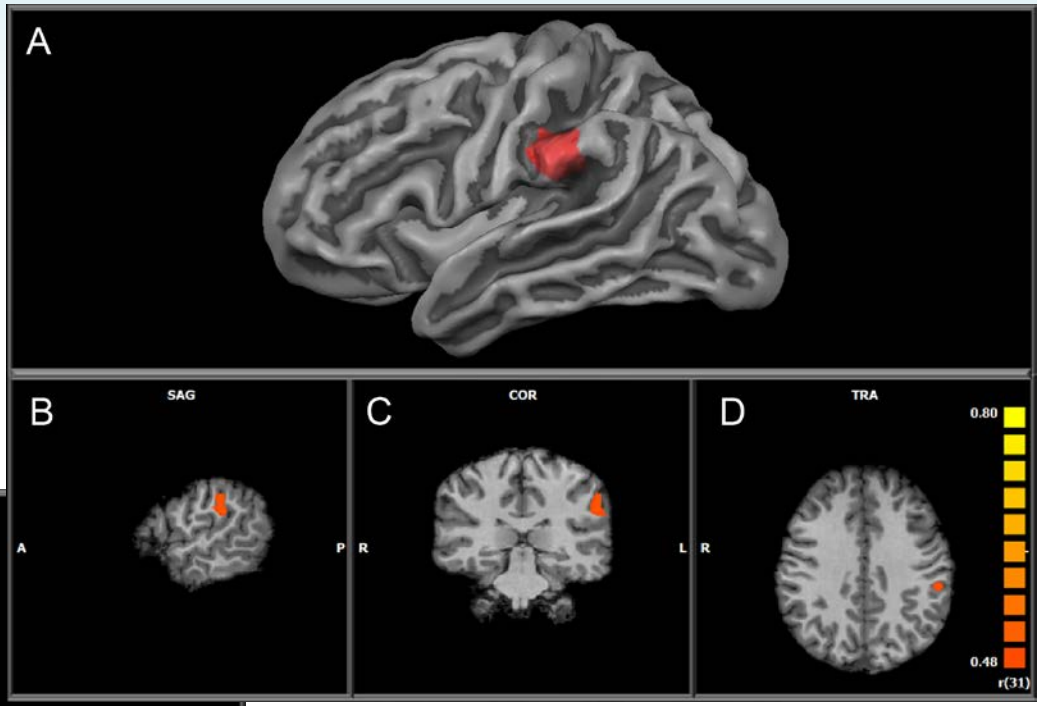
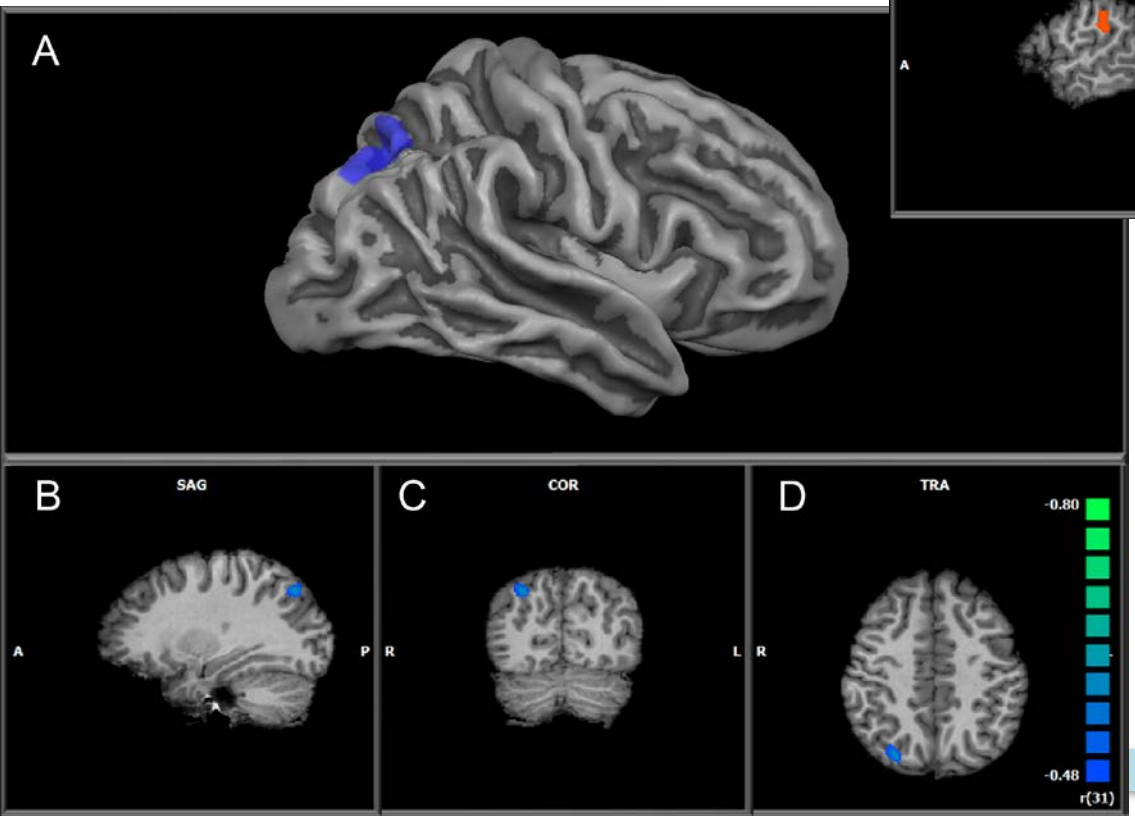
Price et al. (2013) *J Neuro*

PSAT positive correlates with AG/SMG and negatively correlates with IPS

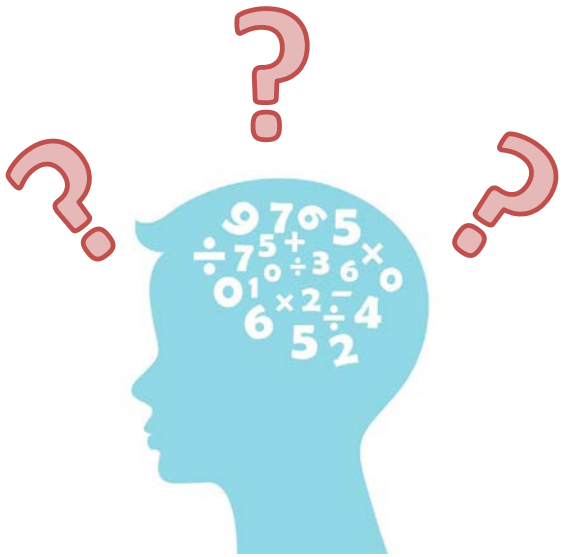




Price et al. (2013) *J Neuro*

How does the brain solve word problem?

- 33-20 = ?
- 阿姨從 33 樓坐電梯往下走 20 樓到書城，書城在幾樓?



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研究方法

- 參與者: 28 位大學生(平均年齡= 22.27, 標準差: 1.84; 年齡範圍: 20.25 - 27.64 歲)
- 數學應用題閱讀作業:參與者須回答題目的描述是否正確。

  - **需數學計算的數學應用題:**
    - 小玉這次數學考 89 分，比上次少 10 分，上次是考 99 分。
  - **不需數學計算的閱讀理解題:**
    - 小玉這次數學考試分數比上次少，小玉上次考的比較差。

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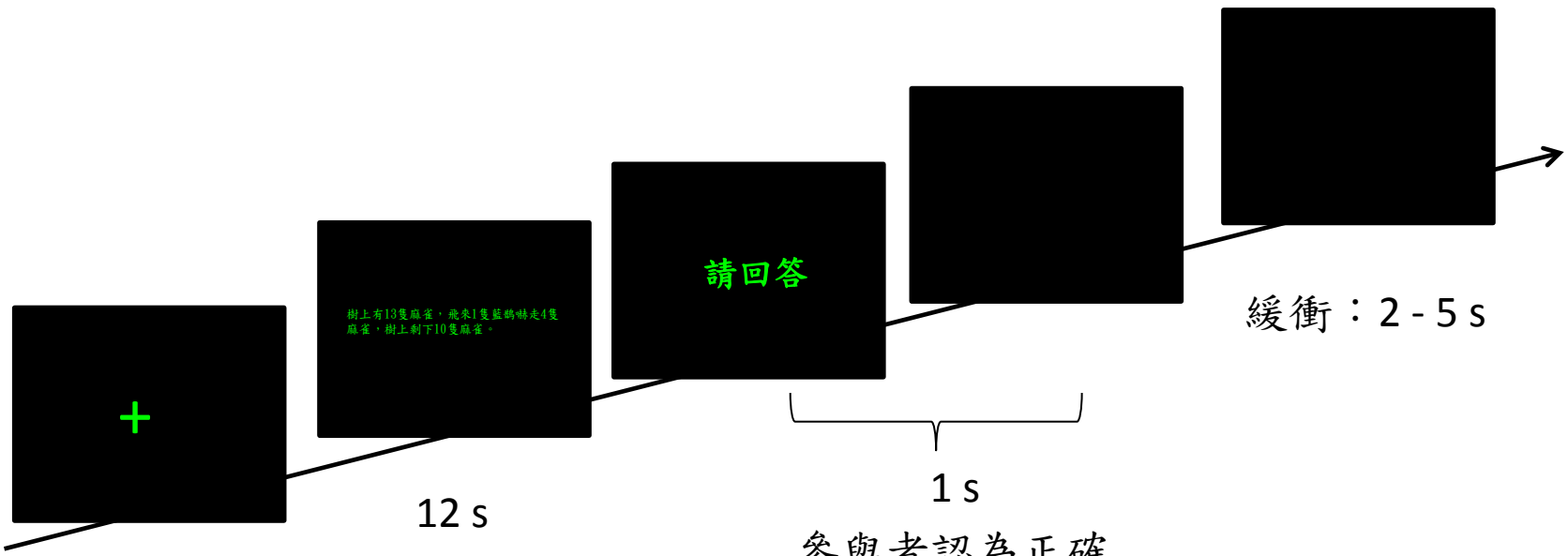
研究方法



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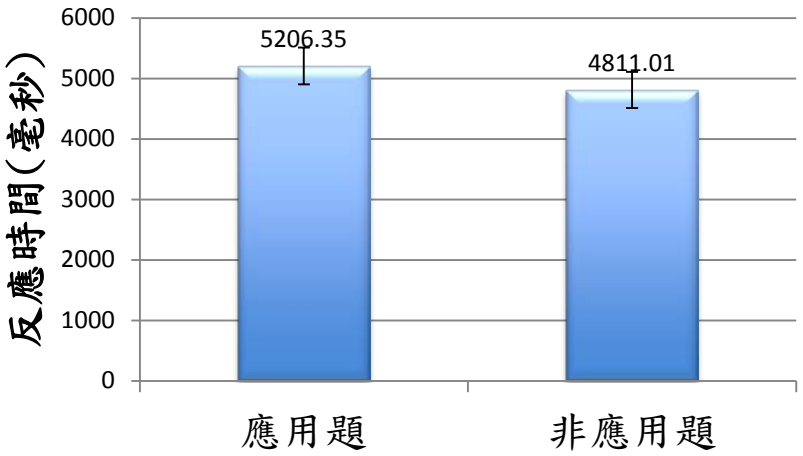
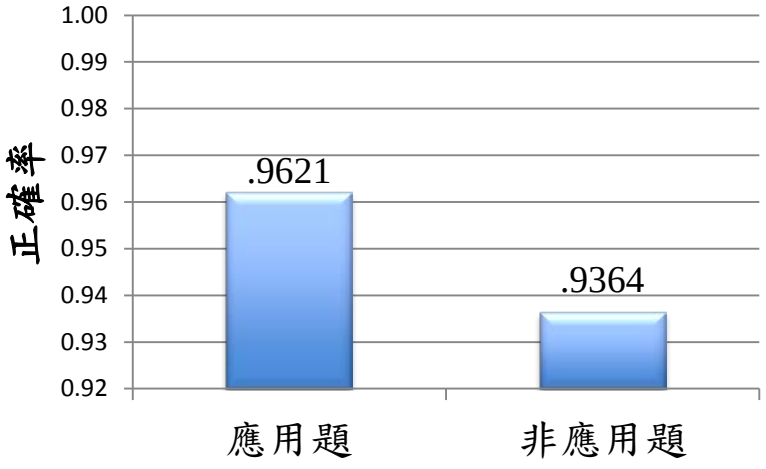
結果

Outscanner performance





		平均	標準差	標準誤	t	p
$(Z_{正確率}-Z_{反應時間})/2$	數學應用題	0.000206	0.000060	0.00	-1.12	0.27
	閱讀理解題	0.000215	0.000057	0.00		

註:這裡的反應時間是每位參與者在MRI實驗結束後，再進行一次應用題閱讀作業的反應時間，該作業不需要等待12秒才回答，而是看完題目知道答案後即可反應。

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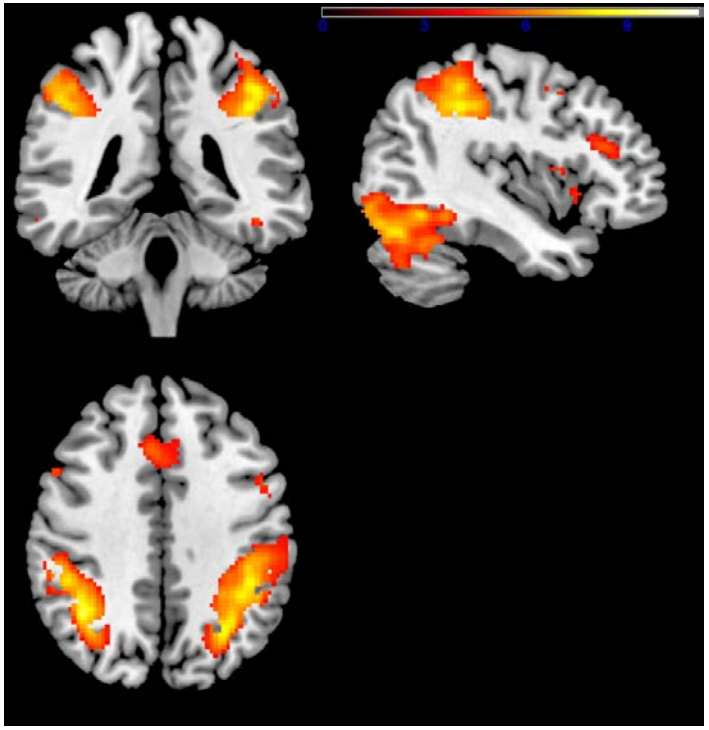
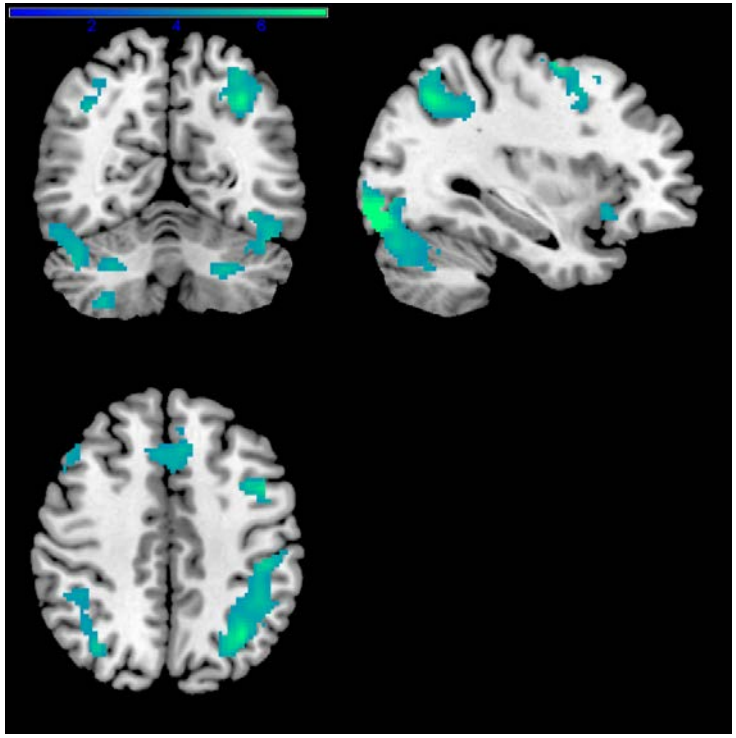
腦造影結果

- 數學應用題



- 閱讀理解題



應用題與非應用題活化的區域, 皆包含視覺區 (V4), 語言區 (Broadman area 44 &45), 以及頂葉

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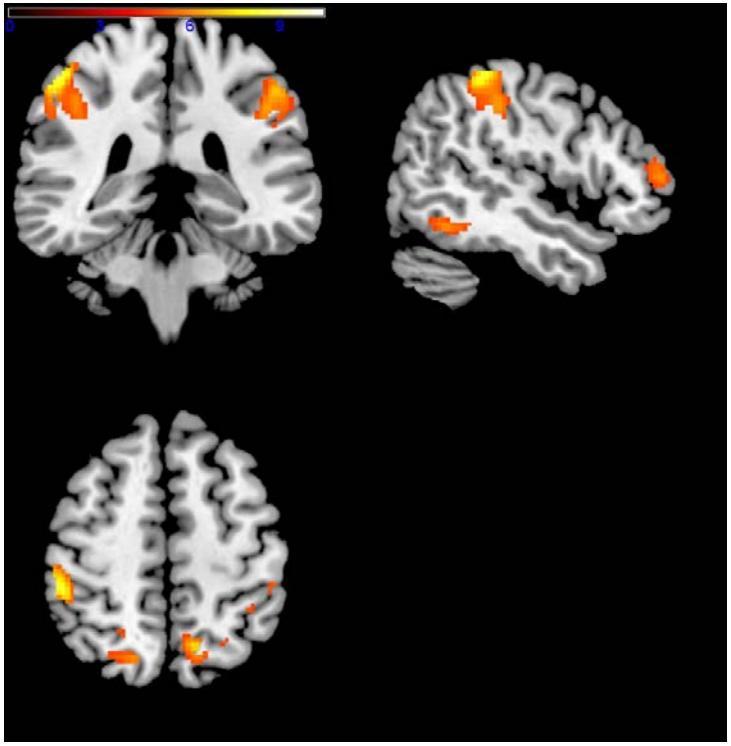
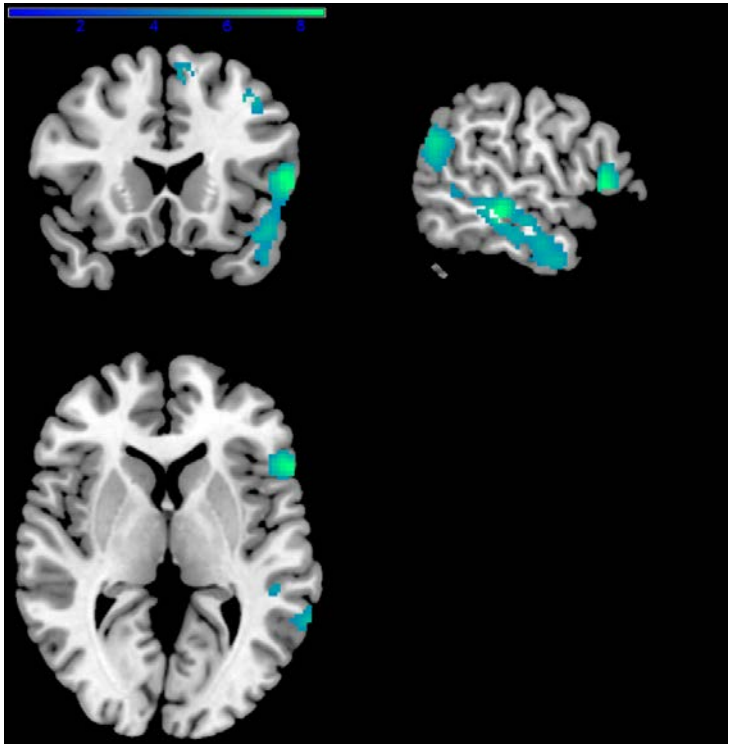
腦造影結果

- 數學應用題 > 閱讀理解題



雙側上頂葉 Right and Left superior parietal lobule, intraparietal sulcus (hIP3 & hIP2)

- 數學理解題 > 閱讀應用題



左側語言區Left inferior frontal gyrus(area 44,45) & angular gyrus

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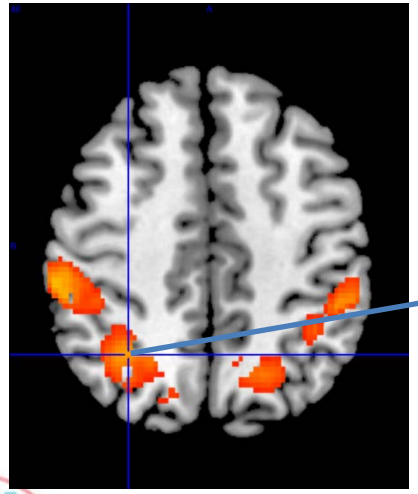
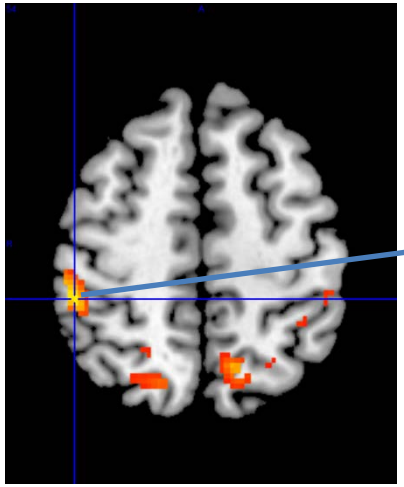
- 數學應用題 > 閱讀理解題

Cluster 1 (1364 vox):						activated %
200.9	voxel =	14.7	% in	right	hIP3	65.3
189.0	voxel =	13.9	% in	right	Area 2	19.9
175.8	voxel =	12.9	% in	right	IPC (PFt)	40.1
81.5	voxel =	6	% in	right	SPL (7P)	12.2
71.9	voxel =	5.3	% in	right	IPC (PF)	8.1
62.5	voxel =	4.6	% in	right	hIP1	27.5
59.4	voxel =	4.4	% in	right	hIP2	45.9



Right hIP3
x = 30 y=-56 z= 46
P (hIP3) = 50%
P (hIP2) = 10%



Area 2
x = 48 y=-36 z= 54
P (area 2) = 50%
P (IPC-PFm) = 30%
P (hIP2) = 20%

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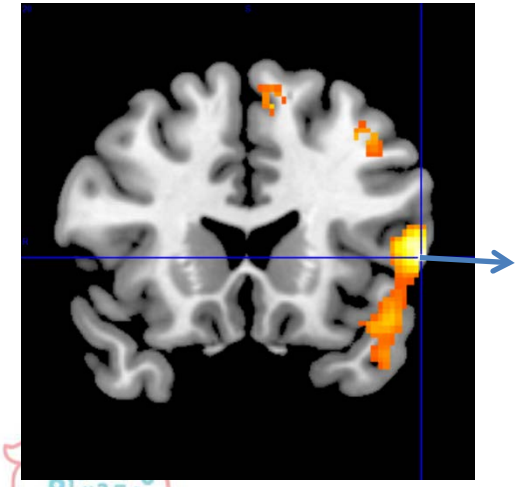
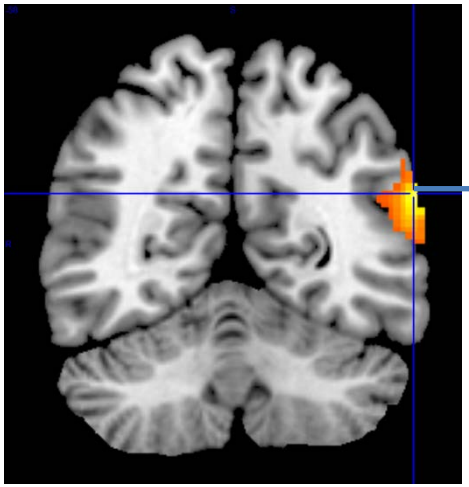
結果

- 閱讀理解題 > 數學應用題

Cluster 1 (2244 vox):						activated %
112.6	voxel =	5	% in	left	Area 45	12.6
87.1	voxel =	3.9	% in	left	IPC (PGa)	11.2
82.5	voxel =	3.7	% in	left	Area 44	7.1
46.5	voxel =	2.1	% in	left	IPC (PFm)	9.2



Area 45
x = -58 y= 20 z= 4
P (area 45) = 50%
P (area 44) = 50%



Left Angular Gyrus
x = -60 y= -58 z= 26
P (IPC) = 20%

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大腦如何發展出數學計算能力？

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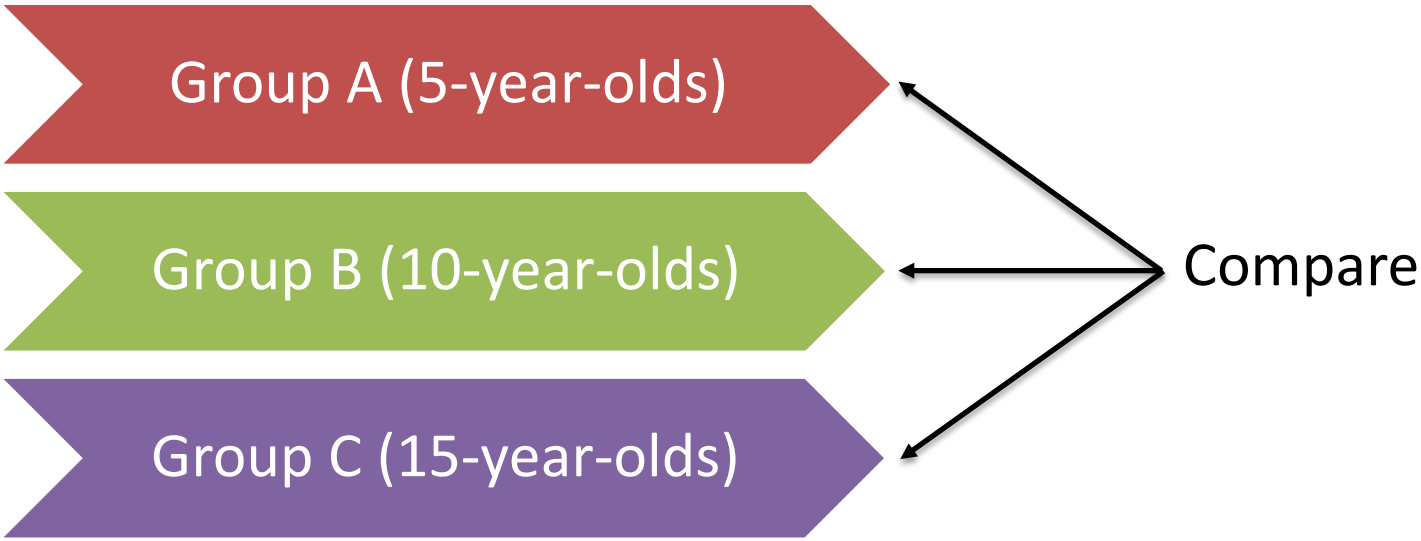
Design for studying brain development

Cross-Sectional Study



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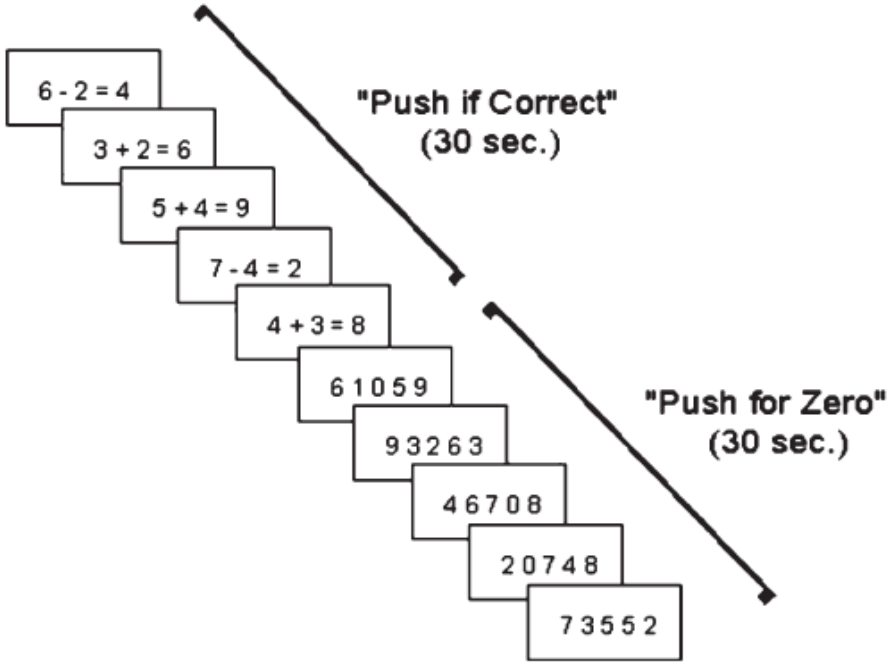
Development of mental arithmetic

Participants

N=16 age (8-19)



Rivera et al. (2005). *Cereb Cortex*

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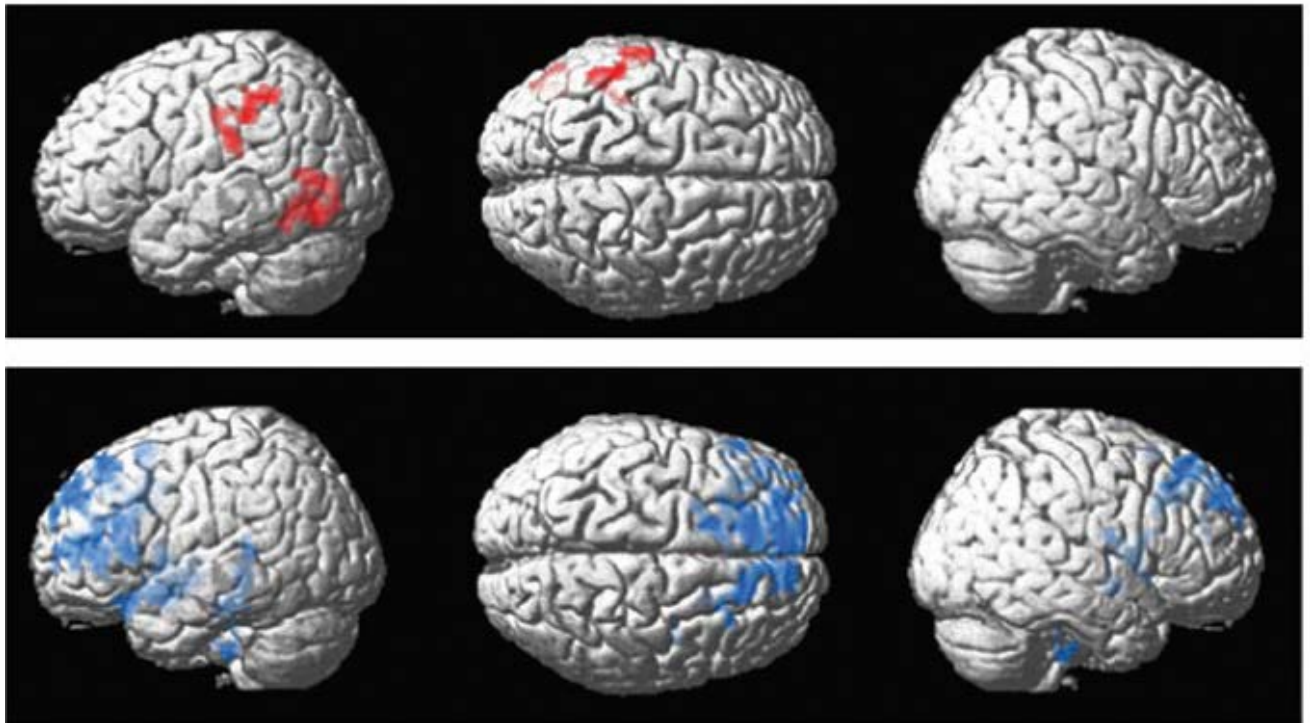
Developmental Change of Mental Arithmetic

Age-related increase of PPC and decrease of PFC



Rivera et al. (2005). *Cereb Cortex*

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Does all PPC subdivisions follow a heterogeneous or homogeneous linear developmental trajectory?

Is there nonlinear developmental change in the PPC?

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Development of mental arithmetic across adolescence

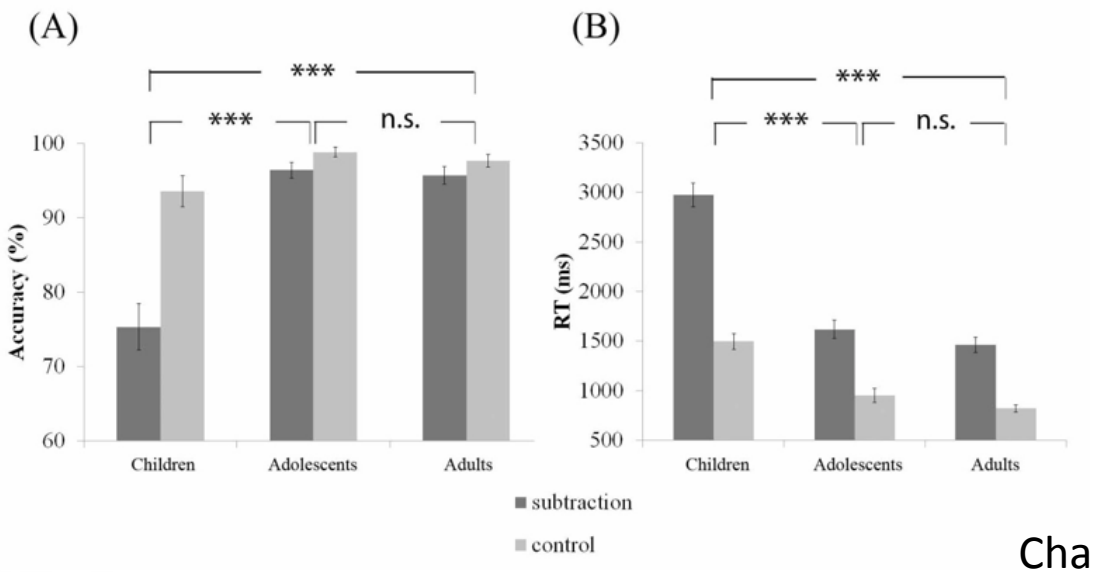
- Participants
  - 25 children (age 7-10)
  - 19 adolescents (age 13-17)
  - 26 adults (age 19-22)

13 – 5 = 9
12 – 3 = 7
9 – 5 = 4

Subtraction

5 @ 1 $ 4
4 % 1 # 3
6 & 1 @ 7

Control



Chang, et al. (2016), *Neuroimage*

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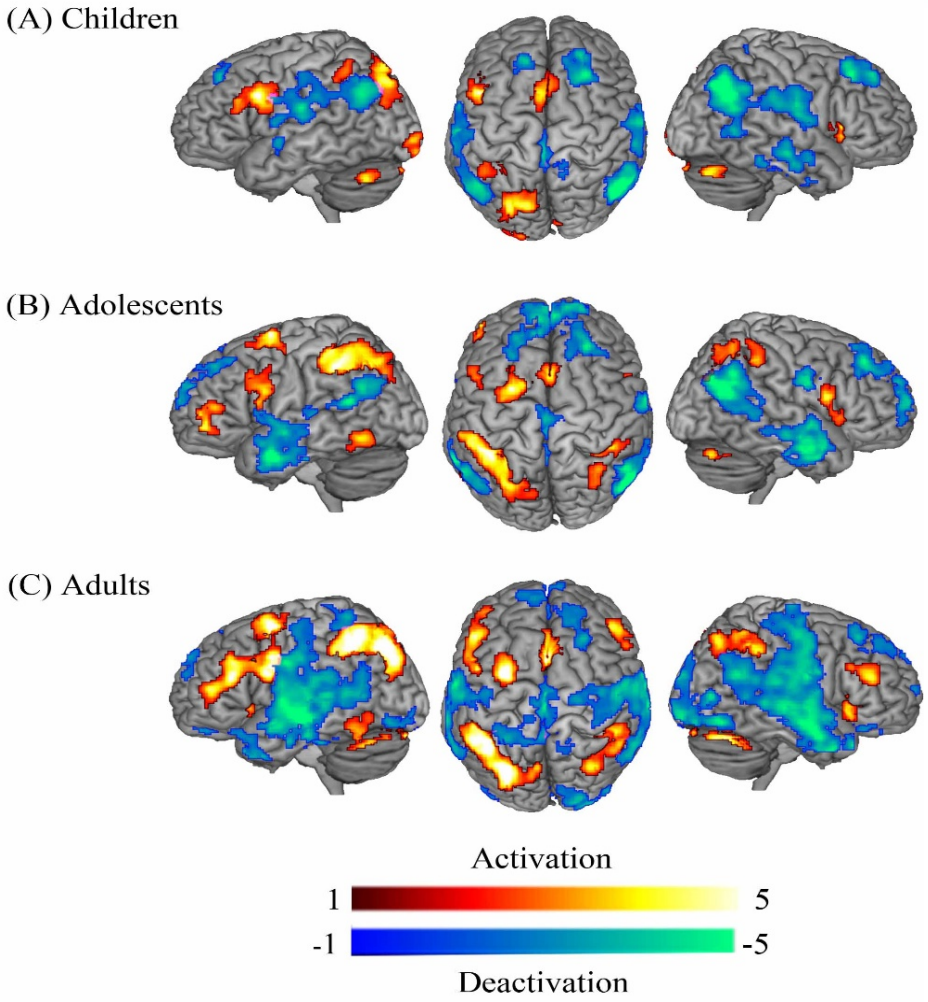
Activations and Deactivations in Children, Adolescents, and Adults



Chang, et al. (2016), *Neuroimage*

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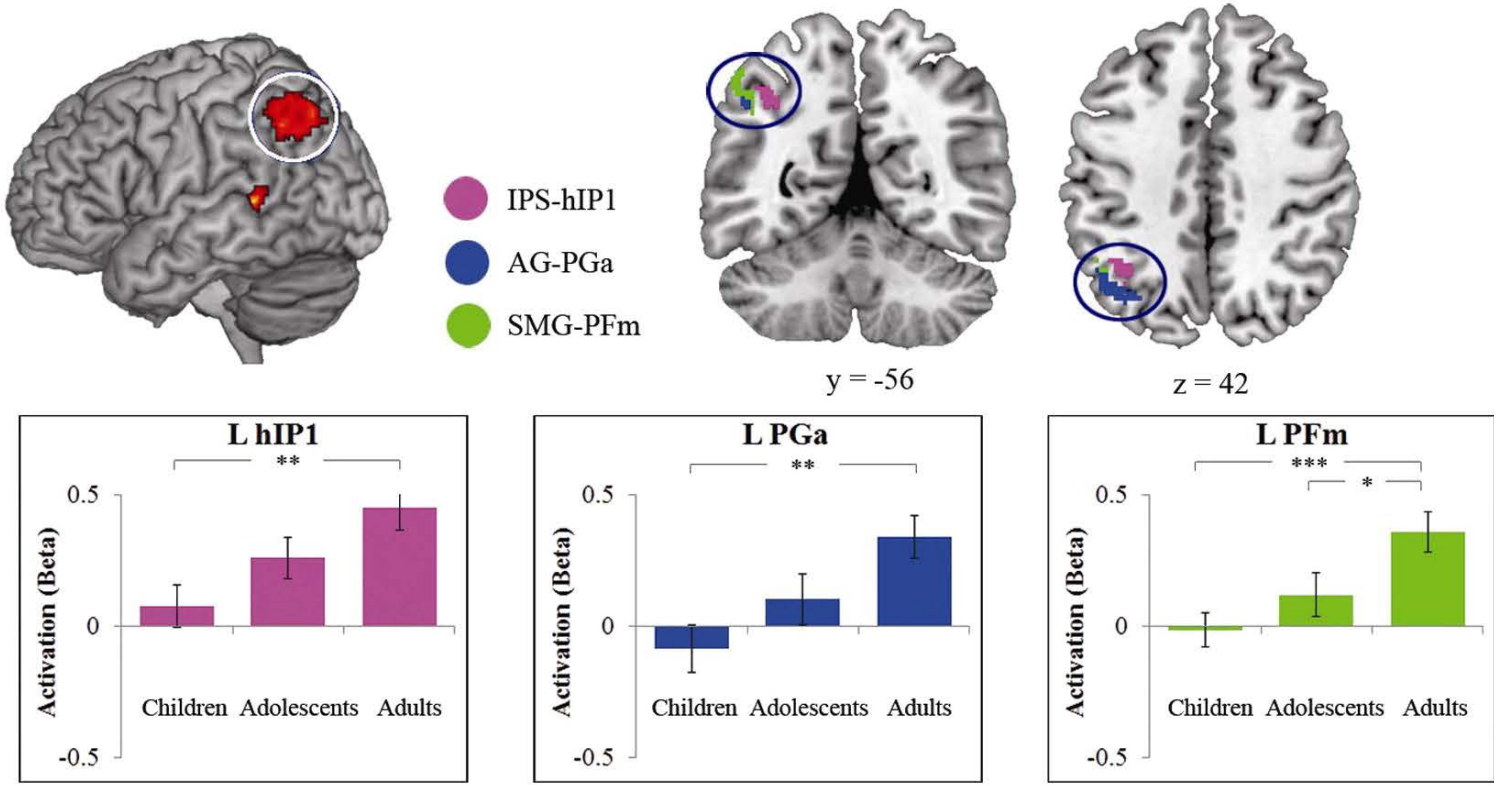
PPC Regions Showing Linear Developmental Changes

A) Linear increases in left IPS-hIP1, AG-PGa, and SMG-PFm



Chang, et al. (2016), *Neuroimage*

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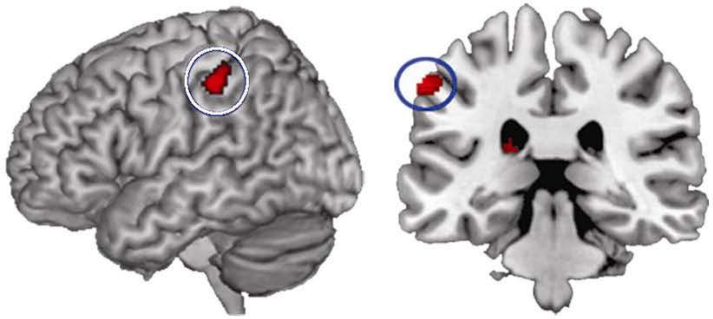
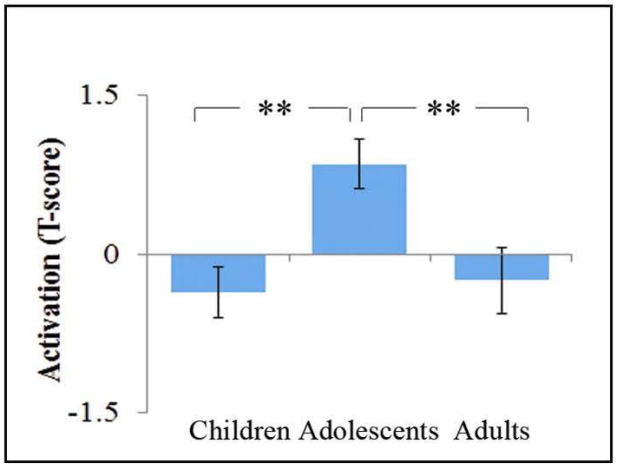
PPC Regions Showing Transient Engagement in Adolescents

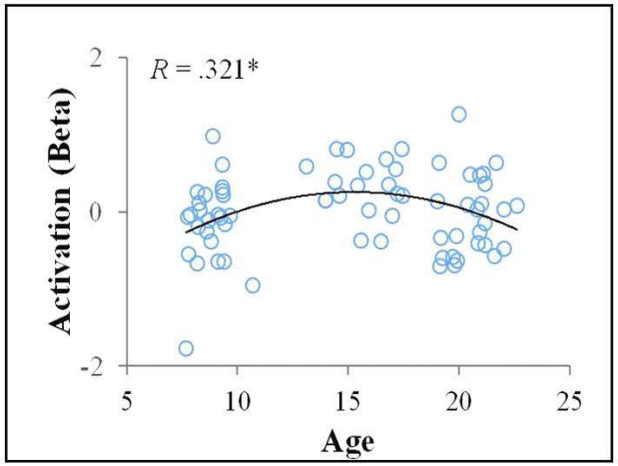
A) Nonlinear increases in left SMG-PF



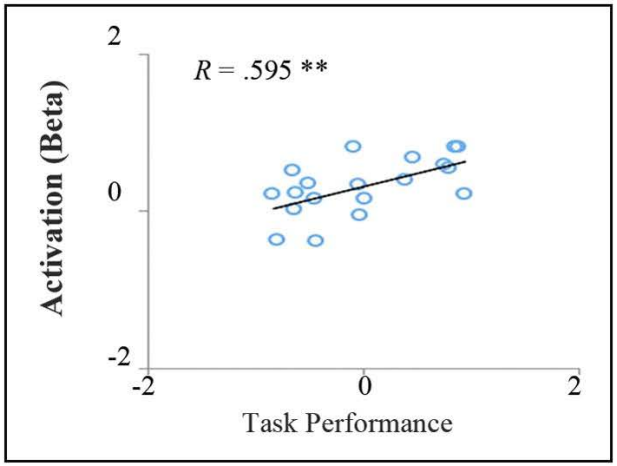
B) Age as a categorical variable



C) Age as a continuous variable



D) Brain-behavior relations in Adolescents



Chang, et al. (2016), *Neuroimage*

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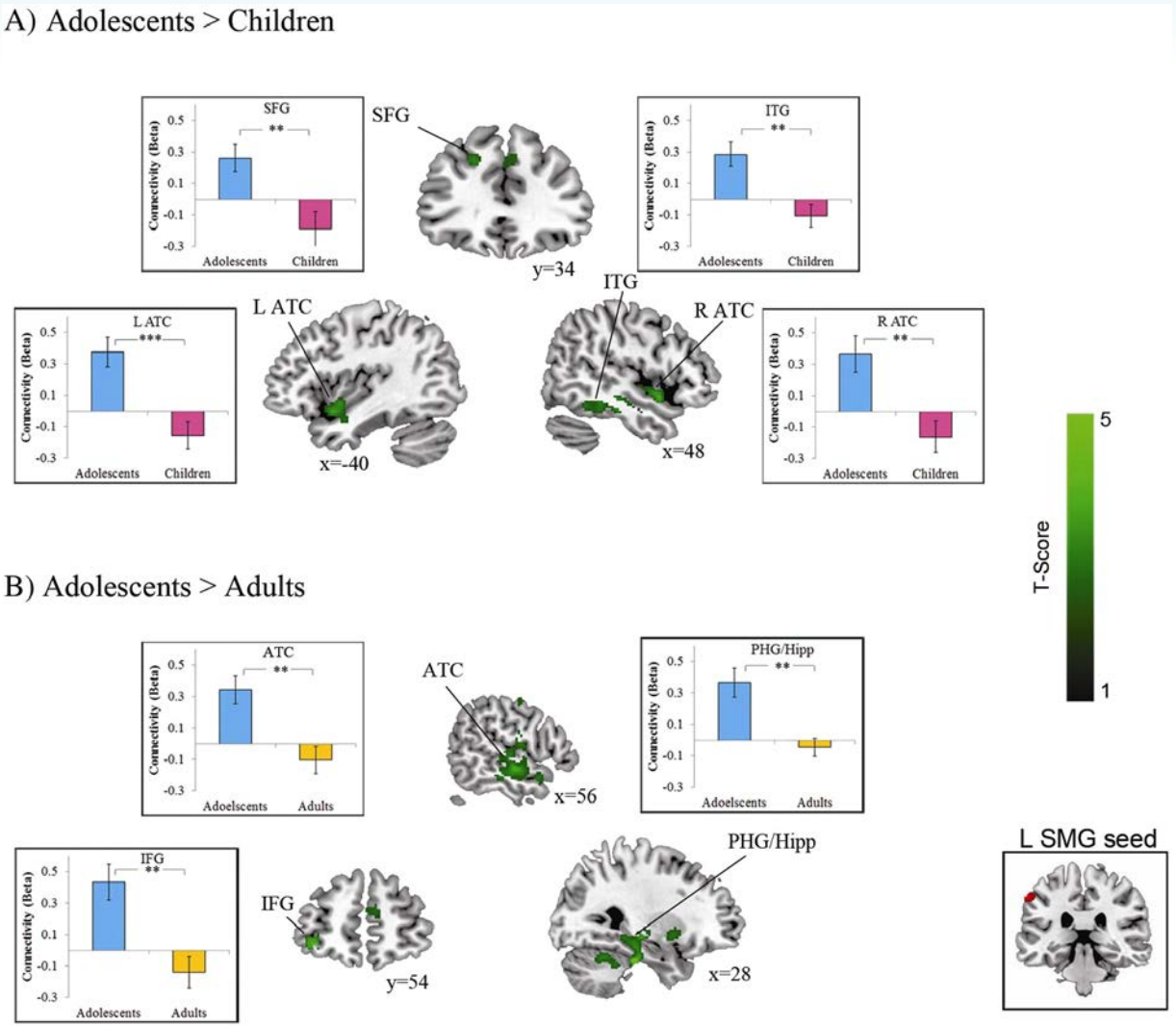
Nonlinear development of SMG connectivity



Chang, et al. (2016), *Neuroimage*

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Neural deficits of mathematical learning disabilities in brain structure and function

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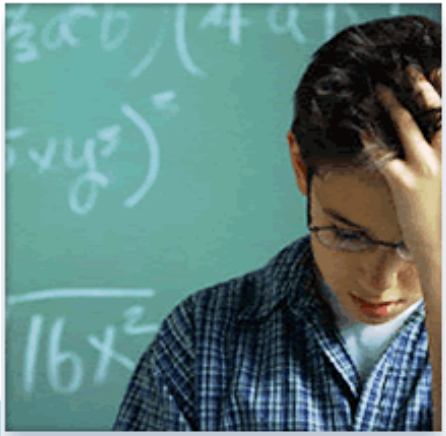
Mathematical Learning Disability (MLD)

- MLD is a specific learning disability affecting the acquisition of school-level mathematical abilities in the context of otherwise normal academic achievement, with prevalence rate of 3-6% (Price et al., 2007).

- MLD children show persistent deficits in mathematical skill.
  - longitudinal study of 140 11-yr old children with MLD (Shalev et al., 2005)
    - After 3 years, 95% of the group still meet MLD criteria
    - After 6 years,
      - 51% could not solvle 7 X 8 (vs. 17% of controls)
      - 71% could not solve 37 X 24 (vs. 27%)
      - 49% could not solve 45 X 3 (vs. 15%)
      - 63% could not solve 5/9 + 2/9 (vs. 17%)



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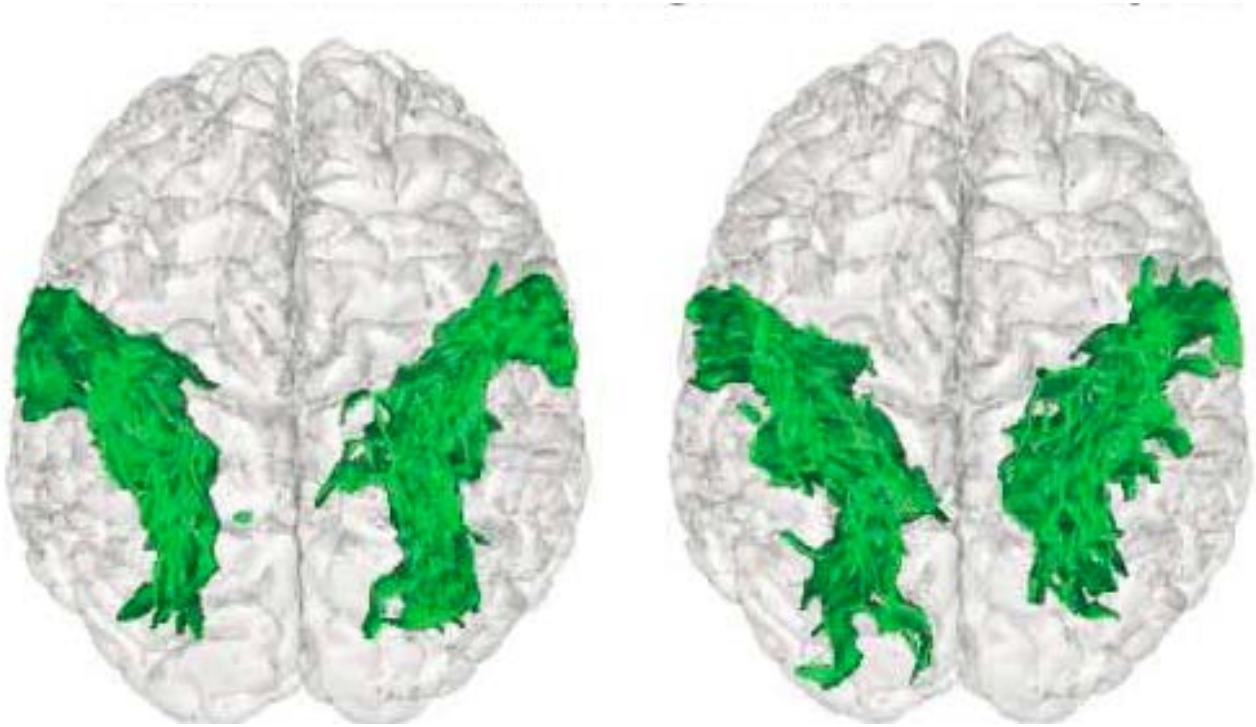
MLD showed reduced size of the right IPS

- 14 genetic-origin MLD and 14 TD participants



TD MLD

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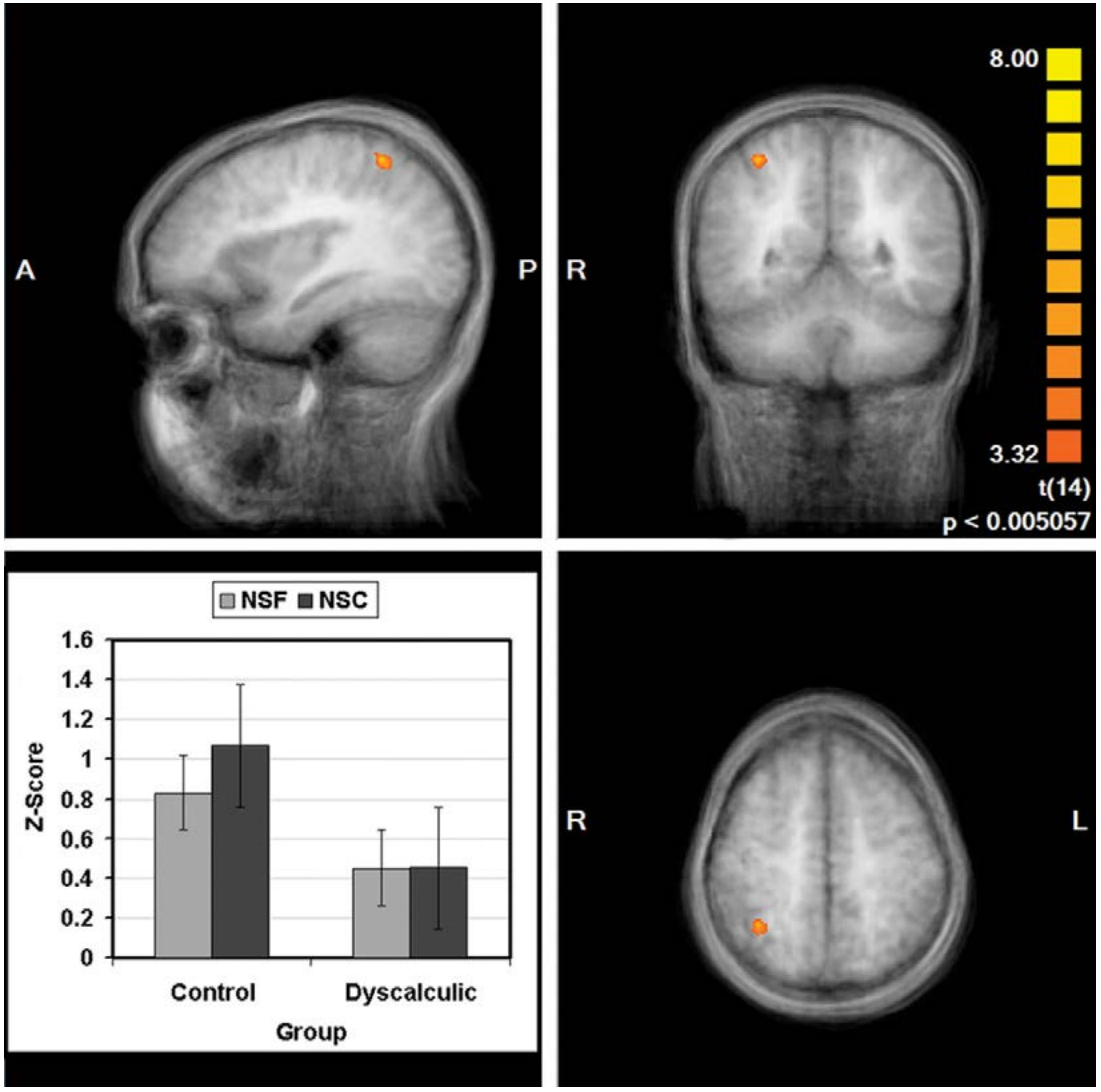
Molko et al. (2003) *Neuron*

MLD showed reduced number distance effect in the IPS



Price et al. (2007)
Curr Biol

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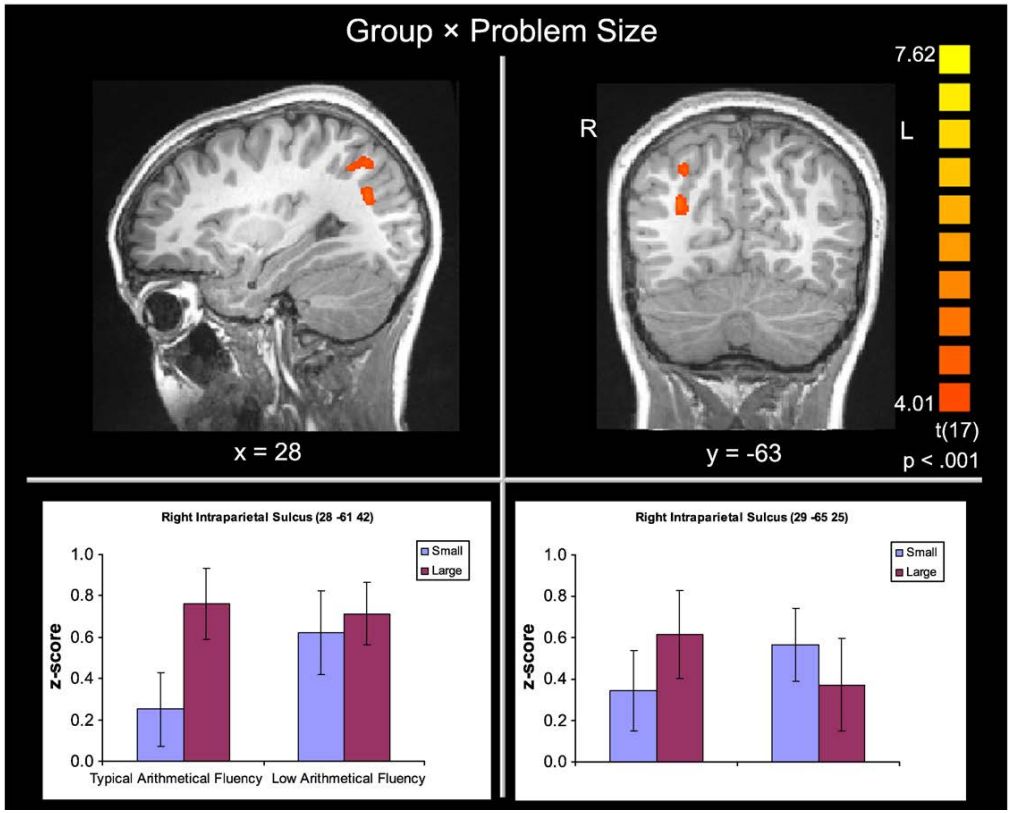
MLD children showed deficits in simple arithmetic problem solving

- fMRI study of complex and simple arithmetic problems
- 10-12 year old children



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DeSmedt et al. (2011) *Neuroimage*

Can the brain function in children with mathematical learning disabilities remediated by cognitive tutoring?

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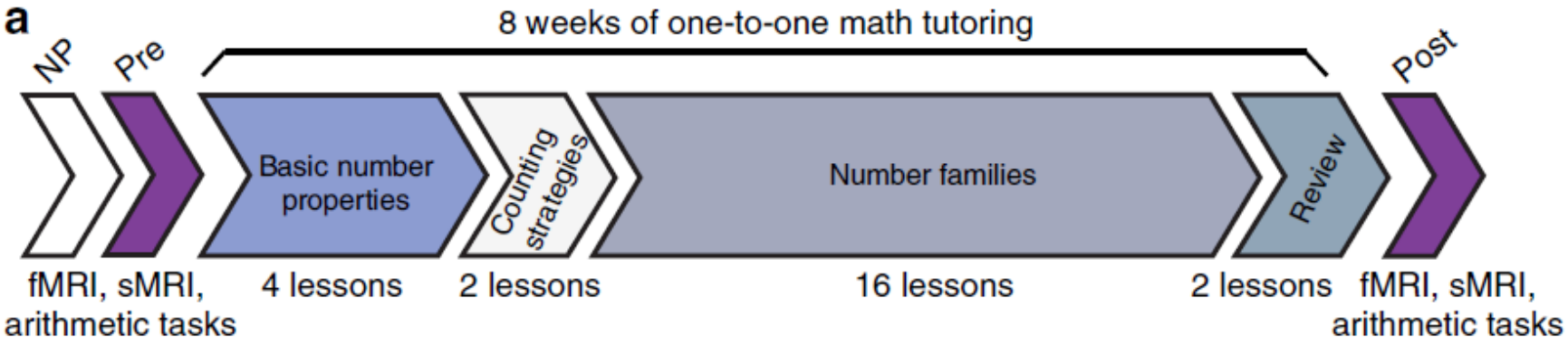

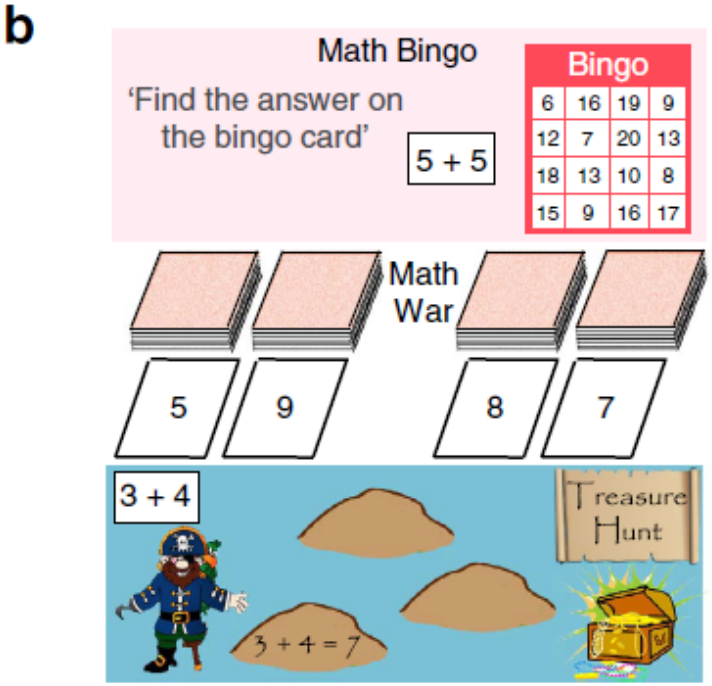

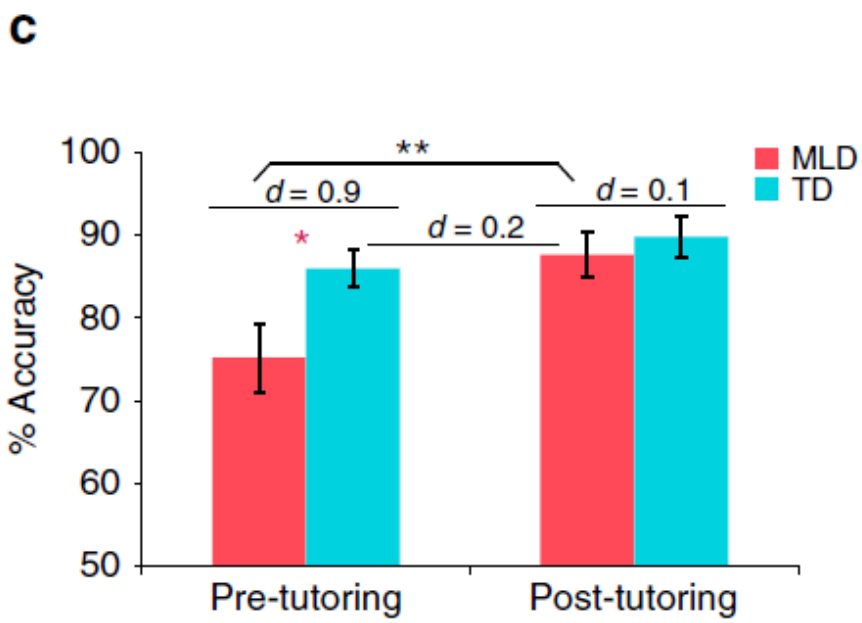

Iuculano et al. (2015) *Nat Comm*

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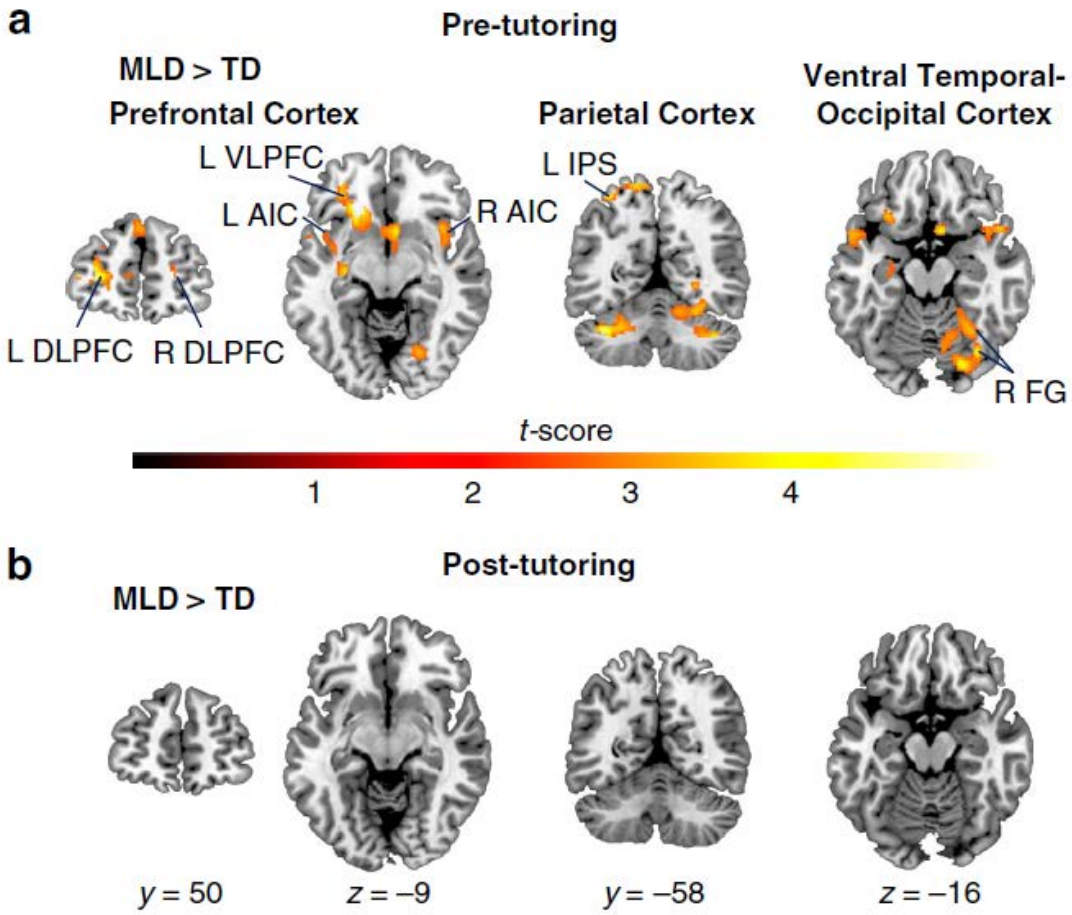
Cognitive tutoring normalizes brain activity in children with MLD



Iuculano et al. (2015) *Nat Comm*

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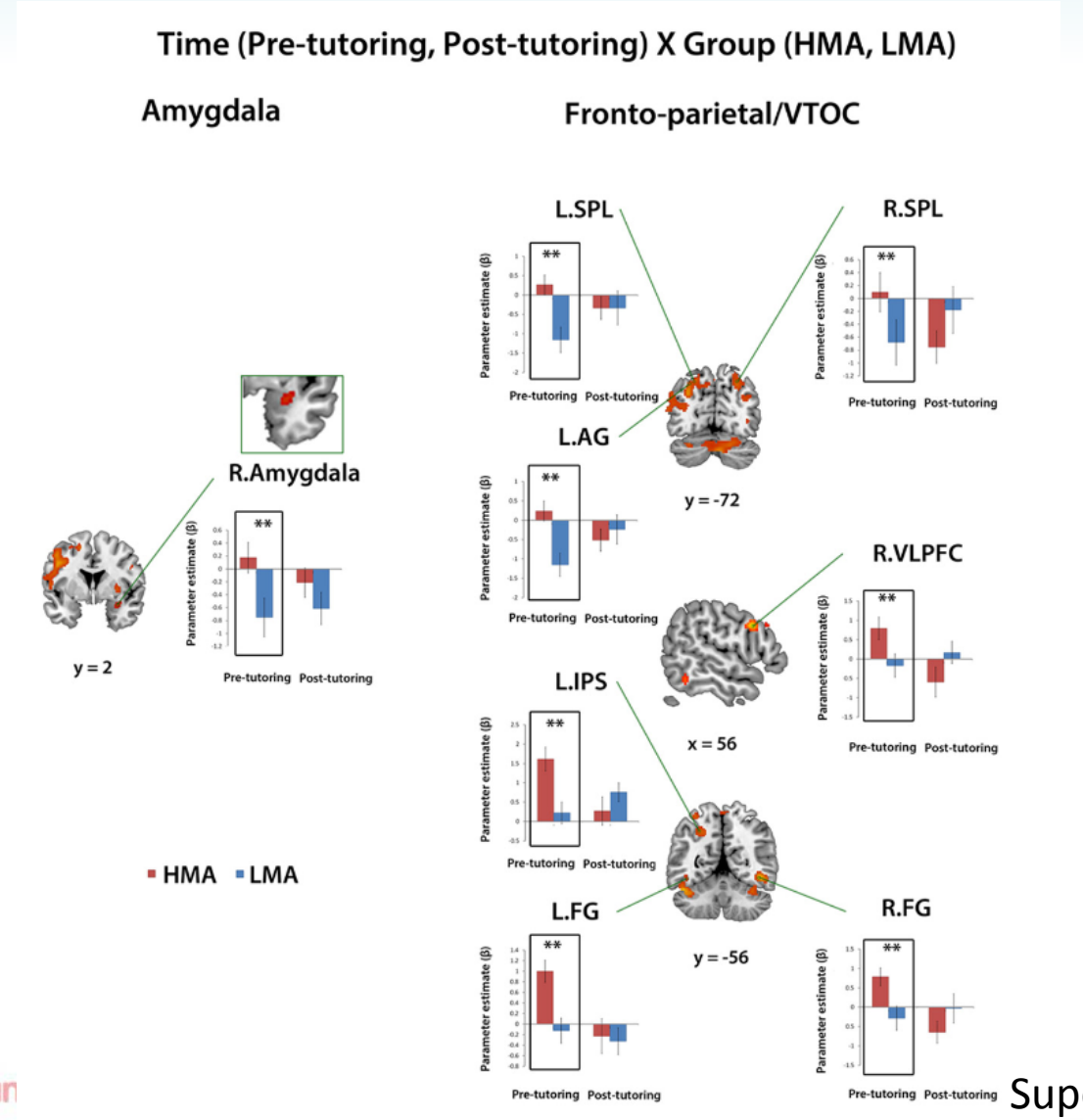
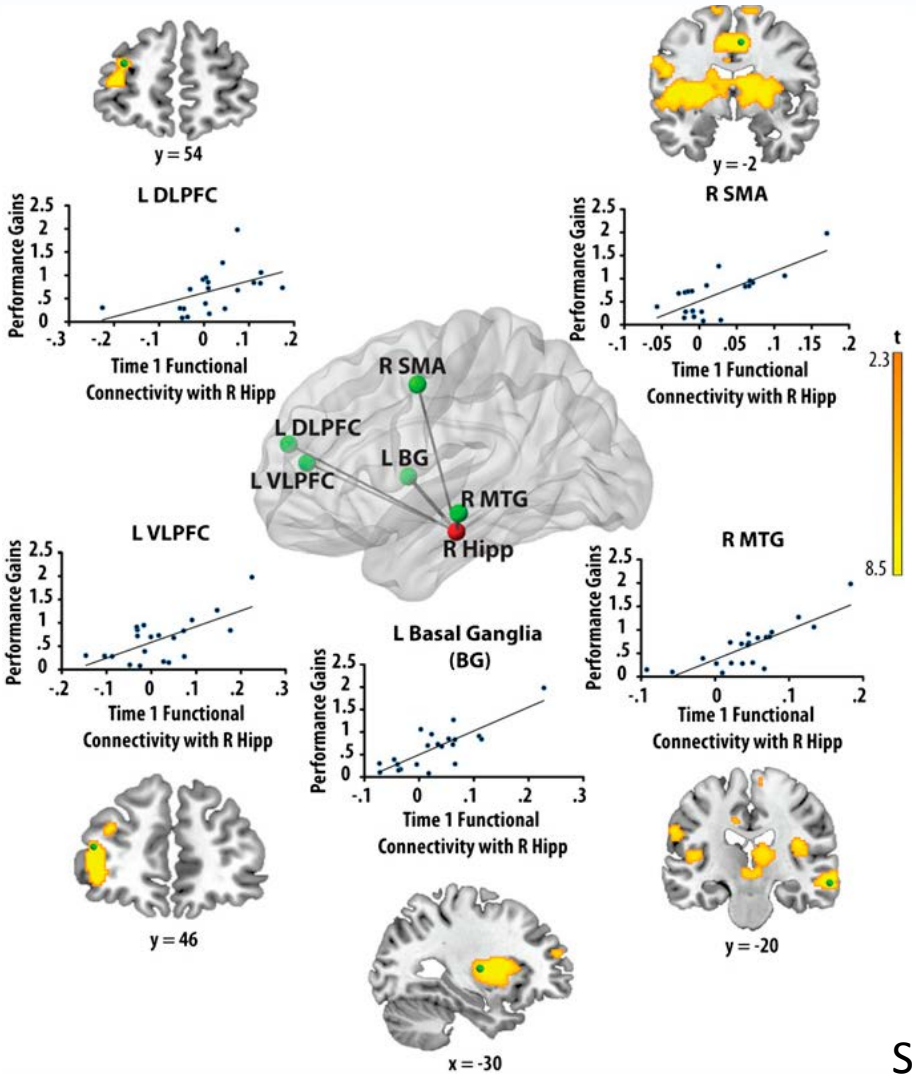
Cognitive tutoring remediates childhood math anxiety and associated neural circuits



Supekar et al. (2015) *J Neuro*

Neural predictors of individual differences in response to math tutoring children



Supekar et al. (2013)

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Summary

- The neuroanatomical function of posterior parietal cortex is mandatorily in involved in mathematical skills.
- Atypical developing in mathematical skill are associated with structural and functional neural deficits.
- Cognitive tutoring induces widespread neuroplasticity by normalizing brain function and strengthening intrinsic connectivity in children with MLD.

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Acknowledgment

Stanford Cognitive & System Neuroscience Lab

政治大學大腦與學習實驗室

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Thank you for listening!

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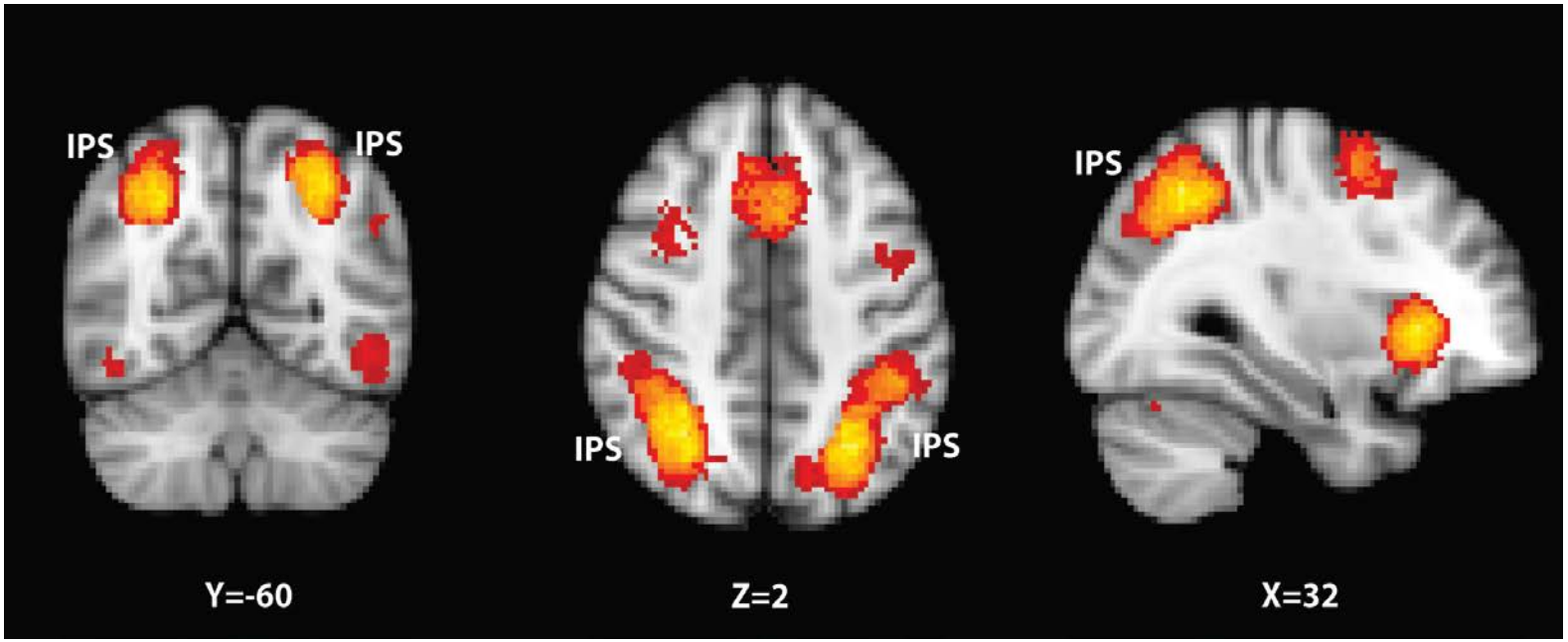
Canonical Brain Areas Involved in Arithmetic Problem Solving



Maps are based on meta-analysis using search term of "arithmetic" in Neurosynth (Yarkoni et al. 2011).

Adapted from Menon (2014)

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